* Please answer all questions in the space provided and submit your solutions on Blackboard by <*the specified deadline*>.
* Please ask me in class if you need any clarifications on the interpretation of the questions.
* Please present your work clearly and concisely so that I can follow your approach and arguments easily.
* You may use reference material, but your submission should strictly reflect your individual effort.
* **There should be no consultations or collaboration**.

**Question 1. Constrained Optimization [12 points]**

A data-processing company processes four types of jobs – A, B, C, and D – for clients. In-house processing costs per job are estimated to be $1, $2, $3, and $4 for job types A, B, C, and D respectively. Each job requires three types of computing resources – CPU, storage, and network services. It requires:

* 1 unit of CPU, 1 unit of storage, and 2 units of network services to process each job of type A;
* 2 unit of CPU, 2 units of storage, and 3 units of network services to process each job of type B;
* 3 unit of CPU, 2 units of storage, and 4 units of network services to process each job of type C; and
* 4 unit of CPU, 3 units of storage, and 5 units of network services to process each job of type D.

Because of contractual obligations the company must process 1,000,000 jobs of type A, 300,000 jobs of type B, 250,000 jobs of type C, and 100,000 jobs of type D in the upcoming week.

Over this time period it has 1,000,000 units of CPU, 1,000,000 units of storage, and 2,000,000 units of network services available.

Limited resource availability prevents the company from meeting the entire demands for all job types through in-house processing alone. The company has two other options to fulfill its contractual obligations: it can out-source some jobs to an external partner, and it can use cloud computing services from a provider. No in-house computing resources are used for jobs out-sourced to the external partner. Nor are in-house computing resources needed when cloud computing services are used.

The external partner charges $1.20 for each job of type A, $2.40 for each job of type B, $3.20 for each job of type C, and $4.40 for each job of type D. The external partner, too, has capacity limitations. It can process at most 100,000 jobs of type A, 20,000 jobs of type B, 20,000 jobs of type C, and 10,000 jobs of type D in the upcoming week.

When cloud-computing services are used, costs per job are estimated to be $1.50, $2.50, $3.60, and $4.50 for job types A, B, C, and D respectively. There are no capacity limitations for jobs processed using cloud computing services.

For your convenience, the information presented above is summarized in the table below:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Job Type** | **A** | **B** | **C** | **D** | **Available** |
| Resources required for in-house processing | CPU units per job | 1 | 2 | 3 | 4 | 1,000,000 |
| Storage units per job | 1 | 2 | 2 | 3 | 1,000,000 |
| Network units per job | 2 | 3 | 4 | 5 | 2,000,000 |
|  | Number of jobs to process | 1,000,000 | 300,000 | 250,000 | 100,000 |  |
|  | Capacity of outsourcing partner | 100,000 | 20,000 | 20,000 | 10,000 |  |
| Cost per unit | In-house cost per job | $ 1.00 | $ 2.00 | $ 3.00 | $ 4.00 |  |
| Out sourcing cost per job | $ 1.20 | $ 2.40 | $ 3.20 | $ 4.40 |  |
| Cloud computing cost per job | $ 1.50 | $ 2.50 | $ 3.60 | $ 4.50 |  |

The company must determine an optimal job processing plan so as to meet their contractual obligations in the upcoming week at minimum cost.

1. Formulate the problem as a linear program (**4 points**)

*Decision variables*:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of jobs processed: | A | B | C | D |
| In-house |  |  |  |  |
| Out sourced |  |  |  |  |
| Cloud computing |  |  |  |  |

*Objective: Minimize cost* =

*Subject to constraints*:

Demand for job type A:

Demand for job type B:

Demand for job type C:

Demand for job type D:

CPU availability:

Storage availability:

Network availability:

Out-sourcing capacity:

*All variables are non-negative*.

1. What is the minimum cost attainable under an optimal plan? (**2 points**)

Minimum cost attainable = $ 3,084,000

1. How many jobs of each type should be processed in-house, out-sourced to the external business partner, and processed using cloud computing services under this optimal plan? (**2 points**)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of jobs | A | B | C | D |
| processed in-house | 900,000 | 50,000 | 0 | 0 |
| out-sourced | 100,000 | 20,000 | 20,000 | 10,000 |
| processed using cloud | 0 | 230,000 | 230,000 | 90,000 |

1. How many units of the available CPU, storage, and network services are used up under this optimal plan? (**1 point**)

|  |  |
| --- | --- |
| Resource | Units used |
| CPU | 1,000,000 |
| Storage | 1,000,000 |
| Network | 1,950,000 |

1. The company has located an alternate business partner (*New-Partner*) that can only process jobs of type A. It can process at most 10,000 jobs of type A in the upcoming week, but the price per job is subject to negotiations. What is the maximum amount that the company should be willing to pay *New-Partner* for processing each unit of job type A? Justify your answer. (**3 points**)

The maximum amount that the company should be willing to pay *New-Partner* for processing each unit of job type A is $ 1.25

*Reasoning*: Decreasing the demand for job type A by 1 job decreases the optimal cost by $1.25; decreasing the demand by 10,000 jobs decreases the optimal cost by $12,500. Hence, if it pays New-Partner less than $1.25 per job (for up to 10,000 jobs) the company can achieve a lower cost than its current optimal cost.

# Question 2: Decision Analysis and Bayes Rule [13 points]

The quality control manager in a chip manufacturing plant has to select one of two available quality control methods. The estimated error rates for the methods are presented below:

|  |  |  |
| --- | --- | --- |
| **Method** | **Type I Error Probability** | **Type II Error Probability** |
| **A** | .05 | .01 |
| **B** | .03 | .03 |

Type I error probability is defined as the conditional probability of rejecting a good lot. Type II error probability is defined as the conditional probability of accepting a poor quality lot.

Historical data suggests that *three* percent of the lots produced in the plant are of poor quality.

**Events**:

: Event that a lot is good : Event that a lot is NOT good

: Event that a lot is rejected : Event that a lot is NOT rejected

**Given**:

Method A: Method B:

1. Based on the information specified above, what is the conditional probability that:
2. A lot rejected by using method A is actually good? (**2 Points**)
3. A lot accepted by using method B is actually bad? (**2 Points**)
4. The organization incurs a cost of $5,000 when it rejects a good lot. It is further estimated that the cost of accepting a poor lot is $ 20,000 (due to liabilities). If the objective is to minimize the expected cost of errors, which quality control method should the manager adopt? Why? (**3 Points**)

Expected Costs = ,

That is:

Expected Costs =

**Expected Cost for A** = = **$ 248.50**

**Expected Cost for B** = = **$ 163.50**

The manager should adopt method **B** since it has lower expected cost.

1. For the current manufacturing process, the a-priori probability that a lot is poor is 0.03. Under what ranges of this probability should the manager to prefer method A? Assume that all other parameters remain as specified in (a) and (b). (**3 Points**)

Let the required probability =

Expected Costs =

=

Thus:

**E[Cost for A**] =

**E[Cost for B**] =

Method A is preferred when **Expected Cost for A** ≤ **Expected Cost for B**

That is, when

*As long as less than 20% of the lots are bad, Method B is better. Thus Method B is a robust choice since the proportion of poor lots is unlikely to be much greater than our current estimate of 3%.*

1. Under what ranges for the cost of accepting a poor lot should the manager to prefer method A? Assume that all other parameters remain as specified in (a) and (b). (**3 Points**)

Let the cost of accepting a poor lot =

Expected Costs =

Thus:

**E[Cost for A**] =

**E[Cost for B**] =

Method A is preferred when **Expected Cost for A** ≤ **Expected Cost for B**

That is, when i.e., when

*Again, notice that Method B is a robust choice since our current estimated cost of $20,000 is much lower than $ 161,666.*